*Serge Salat

Paris/New York 1215-1811-2015. Eight centuries of hierarchies of scale in urban land lots [1]

DOI: 10.14609/Ti_1_15_1e

Keywords: Paris, New York, Fractals, Urban multifractals, Emergence, Platting.

Abstract The development of inner-city Paris (100 km^2) and the grid of the Commissioners' Plan of Manhattan (66 km^2) were recorded on scales of different ages: two millennia for Paris and two centuries for Manhattan. Despite being very different in appearance, with competing feudal powers in a compartmentalised society and competing markets in an open society, land lot hierarchies marked by hierarchies of scale have emerged that are surprisingly similar, as if there were a form of universality at work.

HIERARCHIES, URBAN SURFACES AND UNIVERSALITIES

Hierarchy of scale is the 'signature' of the fractal complexity of urban structures

In urban systems, averages have hardly any sense as values show so many peaks in intensity: 1 square mile (2.56 km^2) of the City of London produces 8.5% of the U.K'.s GDP; 25 km^2 (1 quarter of inner-city Paris) of the 23 wards of Tokyo (600 km^2 and 9 million inhabitants) consume 18% of the city's total energy. The landscape of urban values is not a flat one. The more cities are active, powerful and competitive – like New York, Tokyo, London and Paris – the more the values of wealth, the price of land, the size of elements and the concentrations of network in hubs show clear inequalities. In New York the energy density on the ground (*i.e.* the number of Watts of operational energy needed to make the city work per m^2 of urban land, calculated on the scale of a fiscal land lot) varies by a factor of 100 between the high-rise neighbourhoods of Lower Manhattan and Long Island.

Fractal structures and their classes of universality perfectly describe the mathematical regularities of these extremely unequal and irregular systems like road networks, land lots and intra-urban energy density. The essential notion is a form of symmetry: scale invariance. It is the result of the structural complexity caused by the evolution of urban systems towards complexity due to the effect of their adaptation to external constraints (Salat, 2011). The symmetry of expansion, or scale invariance, can be seen in countless natural phenomena and in living organisms where evolution has favoured scale invariance in structures due to the efficiency of their resilience. Therefore inverse power laws link the different scales: the incidence of an element of size x is inversely proportional to its greater size to an exponent m characteristic of the scaling properties of the system.

[1] Article accepted by Données Urbaines to be published in French in December 2015. The author thanks Denise Pumain, Brigitte Baccaïni, Marie-Flore Mattei and the editorial board of Données Urbaines for their valuable comments and insights.

* President of the Institute of Urban Morphologies and Complex Systems, www.urbanmorphologyinstitute.org

There are a few large elements, an average number of elements of average size and a very large number of small elements (a long tail). The relative incidence of each type is determined by the scaling parameter of the inverse power law.
This profound mathematical regularity emerges in resilient cities, as in all living phenomena. In living cities, it derives from the historic stratification over the course of millennia (Paris) or from powerful market forces (New York). The distribution of elements and connections does not obey the Gaussian theory (concentration around means, Figure 1) but scale-invariant inverse power laws (the Pareto Principle, Figure 2).

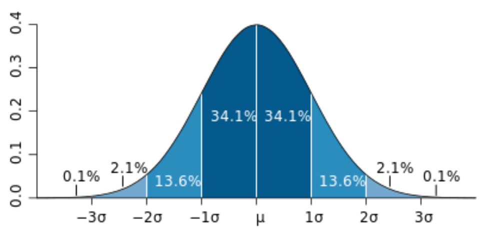


Figure 1 Gaussian distribution, 68% of the values are in an interval of the two standard deviations centred around the mean

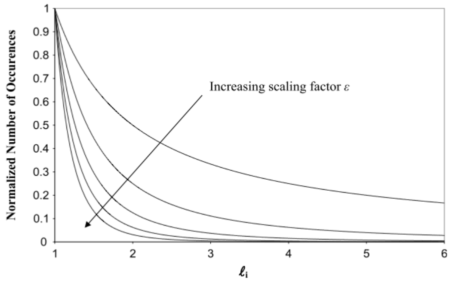


Figure 2 Inverse power law distribution, the higher the scaling exponent the more the gradient between the higher values and the long tail of the weaker values slopes, in other words the more unequal the distribution

The resilience of land lots: long-lasting temporal correlations

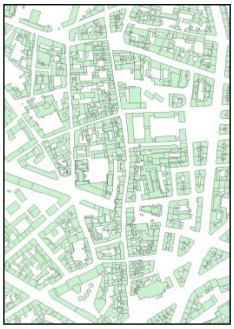
Considering lots is fundament to understanding urban structures as they are one of the most stable elements of cities. Once established, land lots show a marked temporal inertia and temporal correlations over very long time scales. Rome is an example of this permanence. When an Empire falls, several simultaneous phenomena occur: the progressive disappearance of ancient habitation patterns, the reinterpretation of public statues and buildings, especially with temples transformed into churches or broken up and dismantled, and superimposition, so that property structures and old land lots are still both present in Medieval and modern cities.
When London burnt from 2nd to 5th September in the Great Fire of 1666, 13,200 homes, 87 parish churches, St. Paul's Cathedral and the majority of the buildings of authorities in the City were reduced to ashes. The fire cost 90% of the 80,000 Londoners who lived in the City proper their homes. With its system of narrow, tortuous and overpopulated streets, the outline of streets in the City was essentially Medieval. Several plans were proposed for a radically different rebuilding of the City, a movement encouraged by the king. Due to an inability to solve problems of ownership, some of the grandiose schemes for a Baroque city full of squares and avenues never became a reality. The ancient outline was therefore, for the most part, reproduced in the new City, which retains its land lots and Medieval traces.

PARIS: EIGHT CENTURIES OF LAND LOT RESILIENCE

A class of fractal universality

The spatial transformations of Paris in the Middle Ages and those of Hong Kong and Manhattan at the end of the 19th century, follow very different social and economic determinants. However, land ownership shows identical mathematical regularity in size and divisions. We have discovered hidden links between urban phenomena that are apparently very different. This is what physicists call universality classes. They group phenomena that in principle have nothing in common yet, nevertheless, from a certain point of view, adopt a similar behaviour. Later we will see that the density of energy in Manhattan, *i.e.* the quantity of operational energy needed for buildings and human activity (which is a good approximation of the intensity of economic development and economic concentration) belongs to the same universality class as the division into land lots. What do energy density and land lot division have in common? What is there in common between property in the Paris of Philippe II and that of Haussmann, between Paris and Manhattan or between Wall Street and Hong Kong? Geometry. All these divisions are scale invariants, which means that their appearance does not depend on the scale from which one observes them. They are fractals, *i.e.* contrary to the usual lines and surfaces that have dimensions 1 or 2, they have a fractionary dimension, according to the geometry developed by Benoît Mandelbrot to describe this type of objects. The interesting point here is that the attributes of these urban surfaces do not depend on economic or social determinants, at least as regards the forms they adopt. We are dealing with a new form of universality, different from what we are used to in physics and according to which the laws of nature are universal, in the sense that they apply everywhere in the same manner. Here universality is seen as the fact that, despite their different nature, these systems all adopt a similar appearance that has the same fractal dimension.

An analysis of a hierarchy of scale carried out in two very different quarters of Paris shows the universality of the parameters of the hierarchy in the urban Parisian system. The city becomes more complex within itself or it extends and spreads into new quarters with a stable hierarchy of scale. The land lots of the Place de l'Étoile (Figures 5 and 6) seem to be, considered from the point of view of the scaling structure, merely an expanded version of the Rue Mouffetard (Figures 3 and 4), which dates from the High Middle Ages. However, more than seven centuries have passed between these two urban developments and the social and economic mechanisms of bourgeois speculation of the 19th century are very different from that of the feudal system at the time of Phillip II. We find the same scaling factor of -0.5 in the land lots of Hong Kong and those of Lower Manhattan, where international finance is concentrated in the street patterns of 17th century New Amsterdam. The scaling factor of land lots seems to cross eras and continents unchanged. It does not appear to be determined by economic or social factors but by a form of universality in the geometry of the urban surface.



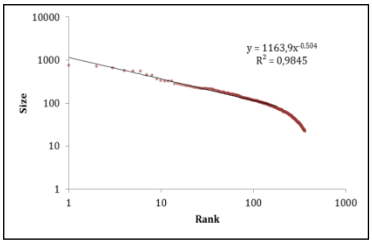


Figure 3 and **Figure** 4: Paris, Rue Mouffetard. In the land lots from the Middle Ages, the incidence of the size of lots (in m^2 and in logarithmic units) is distributed according to an inverse power law with a scaling exponent of -0.5. The rank/size analysis of the land lots is shown here on a logarithmic scale on which the slope corresponds to the exponent of the inverse power law. (Source: Loeiz Bourdic, Institute of Urban Morphologies and Complex Systems, 2014)

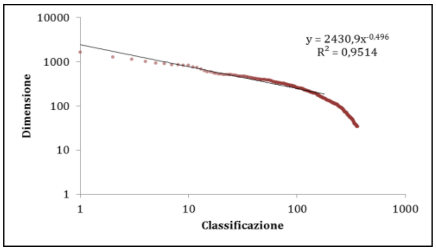


Figure 5 and **Figure** 6: Paris, Étoile. In these land lots from the 19th century the incidence of the size of lots (in m^2 and in logarithmic units) is equally distributed according to an inverse power law with a scaling exponent of -0.5, which indicates the existence of a class of fractal universality. (Source: Bourdic, Institute of Urban Morphologies and Complex Systems, 2014)

In Paris, the class of fractal universality is defined by an exponent -1/2, *i.e.* the inverse of the topological dimension of a surface.

A multifractal palimpsest

Classes of universality are an initial approach, but they homogenise subtle local irregularities caused by the long and dynamic morphological history of cities. Therefore we now have to turn our attention towards a method of description that preserves the details of these irregularities on all scales.

In Paris, from the High Middle Ages until the Revolution, the land was divided between numerous seigneuries. The seigneurs had little by little granted tenures to individuals on their land for which they received annual payment – a rent or *cens* from which came the name of *censive* for the Parisian seigneuries. The Medieval division of land in Paris was the result of the multicellular development of the City from later subdivisions to those of the seigniorial and ecclesiastical *censives* (Noizet *et al.* 2013). It was marked by morphogenetic breaks in the later Medieval city walls, the breaking of the walls of Philippe II for example, which created perceptible asymmetries in the land lots five centuries later in the Napoleonic Land Register of Vasserot from 1810-1836. From the moment in which Philippe II, between 1190 and 1215 (the date from the building of the city walls from which our story starts) built a surrounding wall, the King clearly stated his desire to see the whole enclosed surface occupied by the homes of new inhabitants. This was in fact a new city built from scratch by the Templars while they decided to use their *censives* in the Marais, which was still under-populated. The knights created a new gate in the royal walls (the Chaume Gate) and created a new street between Rue du Temple and Rue Vieille-du-Temple, a centre line through the houses known as Rue de la Porte-Neuve (now Rue des Archives). Six centuries later the asymmetries of the age of Philippe II can still be seen in the Vasserot map of 1810-1836. Hélène Noizet and Étienne Lallau (Noizet *et al.* 2013) have examined the size of land lots on this map along Rue du Temple, Rue de la Porte-du-Chaume and Rue Vieille-du-Temple. They discovered that in 1836 there was a spatial asymmetry linked to a temporal asymmetry in the creation of gates in the age of Philippe II. The oldest two streets show a higher density of small lots (20.3 lots per hectare on Rue du Temple and 15.5 per hectare on the Rue Vieille-du-Temple, opened in 1203) than the lots for housing along Rue de la Porte-du-Chaume opened in 1288 (11.3 lots per hectare with an average size that was twice as small as the lots along Rue du Temple). The asymmetries of the lots carry through the ages the memories of later sedimentation in the urban morphogenesis. These foundations of new burgs in housing lots of *censives* were very numerous and intercalated, both on the right and left banks, in the spaces left between the organic tissue of the first populated centres, with regularly spaced terrains, more often than not along an axial street. These lots with their geometric appearance linked other, older ones with a more complex appearance. It was a result of this constant creative re-writing of the City upon itself, a multiplicity of interlocking fractals, *i.e.* a multifractal. In the mid 80s, mathematicians began to look at functions that seemed highly irregular in some areas and much less so in others, without being able to clearly assign boundaries to these areas: within a fairly regular area one can see irregular areas and reciprocally, this mix belongs to all scales. This ‘multi-scale’ complexity makes us unquestionably think of fractals. Multifractal analysis appeared in physics as a means of understanding and analysing such complex functions and of introducing new quantative parameters to allow their classification. The aim of multifractal analysis is to study functions whose punctual regularity can vary from one point to another.

The first tools for measuring regularity are familiar to all: continuity and derivability at a point. Hölder’s condition introduced a *continuum* between these notions and allows us to detect regularity precisely thanks to a positive real parameter. The notion of singularity is thus introduced in the form of a Hölder condition. Information on punctual regularity is very useful in this form, but some structural information needs to be added. Local analysis is completed by global description of a higher level. This description consists of measuring the fractal Haussdorff dimension of sets of points of the same regularity (‘iso-hölders’) which are fractal sets. The notion of the Haussdorff dimension extends to fractal sets (in which the dimension may be non-integer) and the natural notion of dimension to curves and regular surfaces. The *iso-hölder* sets correspond to different morphological periods in the city. These periods link in the spaces left by previous periods, which they deform without ever completely eradicating. They succeed each other at a more or less rapid, more or less halting, pace - itself a characteristic of a fractal temporality linked to economic developments.

Two structuring morphogenetic axes

The surface of pre-industrial land lots is generally made up of 50 to 100 m^2. The highlighting of land lots of under 300 m^2 in the Vasserot map of 1810-1836, carried out by the ALPAGE project (Noizet *et al.*, 2013), reveals a greater land lot density (11 lots per hectare) on the right bank, the favoured axis during Medieval development, than on the more rural left bank in the Middle Ages, (with 8 lots per hectare). The graphic extraction of smaller lots in Napoleonic Paris reveals, as a result of its Medieval past, a fractal image of a spread (Figure 7) clearly orientated along two perpendicular axes creating an angle with the geographic east.

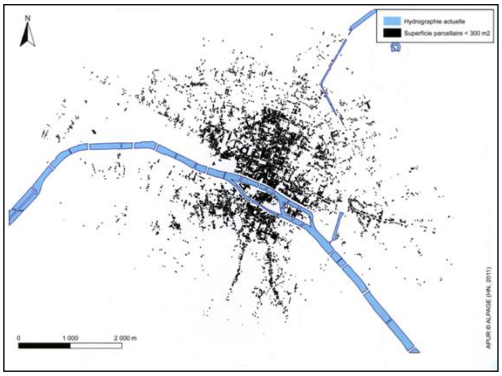


Figure 7 Extraction on the Vasserot map (1810-1836) of land lots with a surface area of under 300 m^2. A fractal set that draws Paris as it was five centuries earlier. Source: APUR, ALPAGE, 2011

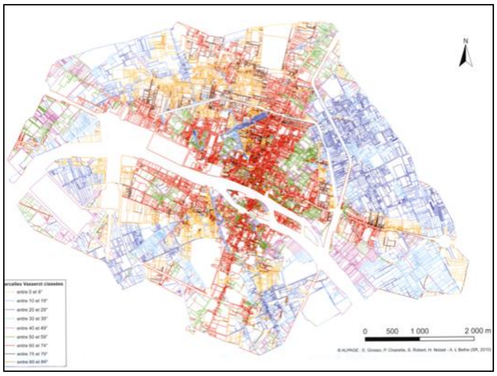


Figure 8 A map of the orientations of Vasserot lot segments (1810-1836) as well as those of the archaeological structures of Paris. Source: ALPAGE, E. Grosso, P. Chareille, S. Robert, H. Noizet, A.L. Bethe, 2010

The orientation of the lots confirms the distribution of lots along two dominant perpendicular axes (Figure 8). The dominant orientation situated between the 60th and 74th, to east of the map, represents alone 36% of the total segments of the Vasserot land lots. It rests on two extremely morphogenetic axes, *i.e.* that can generate and transmit forms: the alignment formed by Rue Saint-Martin and Rue Saint-Jacques together with the Seine (Noizet *et al.* 2013). This orientation has long been identified by archaeologists as the dominant one in ancient times. The morphogenetic axis of the ancient orthogonal grid is the Saint-Martin – Saint-Jacques alignment, which in part corresponds to the *cardo* of the ancient foundation and lies on ancient islands that once existed along the course of the Seine. This orientation is also dominant in the street network that existed at the end of the 14th century. The Middle Ages therefore played an essential role in the resilience of the main Roman orientation, in its spread mainly on the right bank (Noizet *et al.* 2013).

NEW YORK: THE CHESSBOARD AND THE BREAKING OF SYMMETRY

Squares on a chessboard

On Manhattan's chessboard a game, that lasted eight centuries in Paris, was played in a single generation. Everything began with a collapse – that of the city's finances in 1776 after the War of Independence. Towards the end of the American Revolution of 1776, the fundamental elements of Manhattan were virtually unchanged since the discovery of the island by Hudson, with the exception of a small town of 32,000 inhabitants, to the south of the island.

This ruined town decided to sell its public land, about 5 km² of rocky ground that was completely undesirable, in the middle of the island, where the greatest wealth in the world is today concentrated.

Figure 9
British Headquarters Map of New York, Long Island, Hudson River, East River showing British and American fortifications, ca. 1782, The National Archives, United Kingdom, MR 1/463

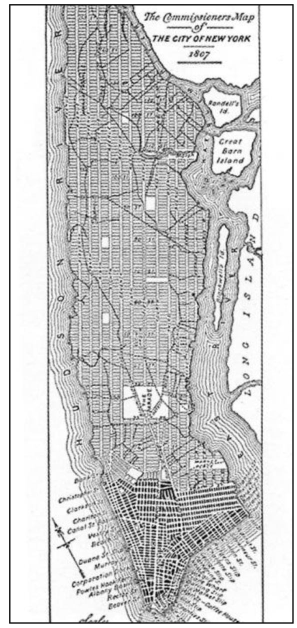


Figure 10
Commissioners' Map of Manhattan (1807-1811) placed on top of a grid of apparently uniform rectangles on the extremely uneven territory of the island

The grid of Manhattan (Figure 10) was first and foremost a format to facilitate the sale of terrain and land development. The grid emptied the real island of any local or topographical feature. It became a pure, abstract surface. The hills were erased in an irresistible drive to develop the avenues northwards, which temporarily left the homes of the first colonists as if suspended in the air.

The grid transformed the island into a pure concept: that of an infinitely versatile, combinatorial land market, open to endless speculation, ceaselessly recreating itself, with constantly rising land and property values. In 1807, New York's total property value was 25 million dollars. This value had risen to 2 billion dollars by 1887: 80 times as much!

This apparently uniformand isotropic grid, which erased all differences, would give rise to an incredible diversity and hierarchical structures: neighbourhoods with identities as different as the Washington Square of Henry James, Soho, Tribecca, the Upper East Side, or Woody Allen's Brooklyn. How could this diversity, variety and hierarchy of scale emerge from a grid? Thanks to subtle differentiations, to the breaking of symmetry here, as in physics, creating structures that would then continue to become more complex.

Firstly, the grid of Manhattan contains two metric patterns that create variety. One of these is created by the width of the streets: 30 metres for avenues running north-south, 20 metres for standard transversal streets, with 15 major transversal streets, 30 metres wide at irregular intervals. The second pattern is a result of the variety of dimensions in the city blocks.

All the blocks are 60 metres wide from north to south, but their length from east to west varies, diminishing from the centre towards the coast. From 3rd to 6th Avenue the blocks are 280 metres long. Towards the east they shrink to 189, 198 or 195 metres long. Towards the west they shrink uniformly to 244 metres in length.

The grid also contains a hierarchy in the topological properties of the streets.

The theory of graphs defines the continuity of a street as the number of street segments between the intersections.
It defines the connectivity of a street as the number of other streets to which it is connected. As the avenues in Manhattan are connected to 155 streets, while the streets are only connected to about 11 avenues, there is an important variation of topological scale between the avenues and the streets.
This initial breaking of symmetry was enough to lead to enormous growth in complexity, creating a subtle and complex form of order, capable of both stability and development, where in creating new structures one adapts to constantly changing conditions. Manhattan is not a crystal.
Its order, unlike that of Le Corbusier's *Ville Radieuse*, is not crystallised in three dimensions. Its shape is only defined on a map by a grid.
The transfer of building permits creates an almost endless freedom for development in the third dimension. It is a chessboard on which the movement of pieces allows one to play an infinite number of games.
Who are the players? They are the human beings who interact every day with the physical forms of the city, with their endlessly reconfigured interactions, exchanges and transformations of money, of symbolic signs, of matter and energy, which ceaselessly increase the quantity of algorithmic information in the urban system. Simple calculations show that the size of the elementary squares on the chessboard of New York or Barcelona – the land lots – is 3000 times smaller than the super blocks of the Le Corbusier's *Ville Radieuse*. This Figure leads to vertiginous differences in terms of connectivity and variety in the pathways of the urban structure, *i.e.* in terms of potential for interaction and in terms of the diversity and the variety of possible localisations.
Connectivity, diversity and variety, under the effect of combinatorial mathematics, increase almost endlessly, as the urban mesh becomes very fine, due to the factorials that express the numbers of possible positions and connections between the pieces on the chessboard squares.

How the property market creates a hierarchy of scale

In this immense space of configurations, human activity does not increase entropy to a state of chaos.
It is superimposed on the uniform grid and creates another structure, an order that is much more flexible and moving, but showing elements of permanence and stability.
This second order still constantly transforms its organisation. It is one of complexity. In Manhattan, the blocks were initially sub-divided to sell terrains in identical 205 m² lots, which, under the influence of market forces, began to be joined to create a gigantic combinatorial mosaic of around 300,000 lots.
The land lots of Manhattan, in their extraordinary fractal diversity, enclosed in a Euclidian grid, representing the perfect hybridisation of order and emergence, were essentially created in 1835, a generation after the Commissioners' Map. The property market in Manhattan was a formidable temporal accelerator that caused the differentiation and emergence of scale-invariant structures. One example is Charles Moore's strategy for developing his large domain, which would become the village of Chelsea (Figure 11).
The breaking of symmetry created by Chelsea Square, the church and the park, caused a stream of differentiations in the size and value of lots depending on whether they were located near to or far from the church.

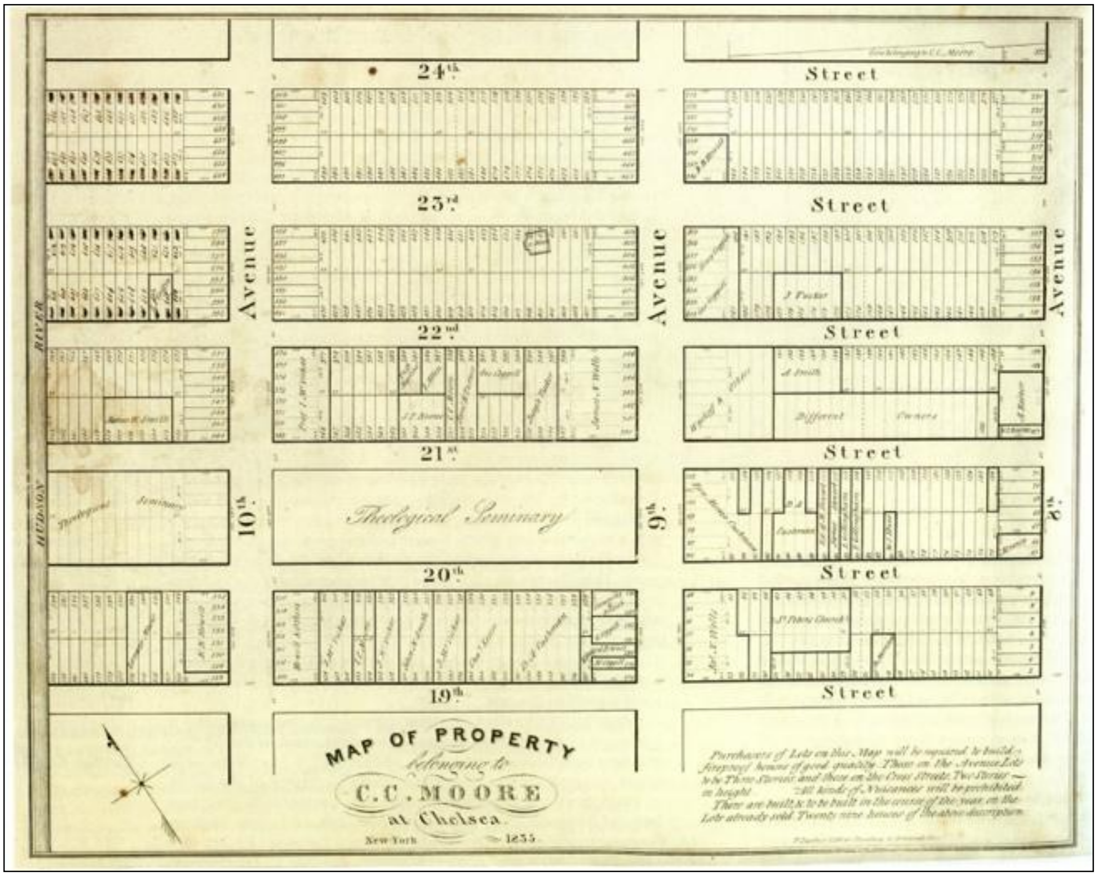


Figure 11 Charles Moore's property map from 1835, which would form the neighbourhood of Chelsea. Collection of the New York Historical Society. As early as 1835, his map shows the strategies of large

Moore centred the village of Chelsea around Chelsea Square, made up of two blocks of Manhattan, which he donated, in 1819, to the Episcopalian church. In 1835, the lots around the square whose property value was much higher, were joined together by rich purchasers to create larger lots.

In 1820, Moore had valued his property at 17,000 dollars. Its value was estimated at 350,000 dollars in 1845 and 600,000 dollars in 1855, a multiplication of 35 in as many years.

The differentiation and asymmetry of land prices emerged very quickly within the grid.

In 1860, properties along Fourth Avenue were valued at between 3,500 and 10,000 dollars, while those along Madison Avenue were valued at 18,000 dollars and 55,000 dollars near Madison Square.

In a scale-invariant morphological field like the grid and the Manhattan land lots, the form and price of each element is influenced by its interactions on different scales with all the other elements.

When the results of these interactions create a form, it is never fixed or symmetrical. It exhibits a degree of plasticity that allows it to evolve.

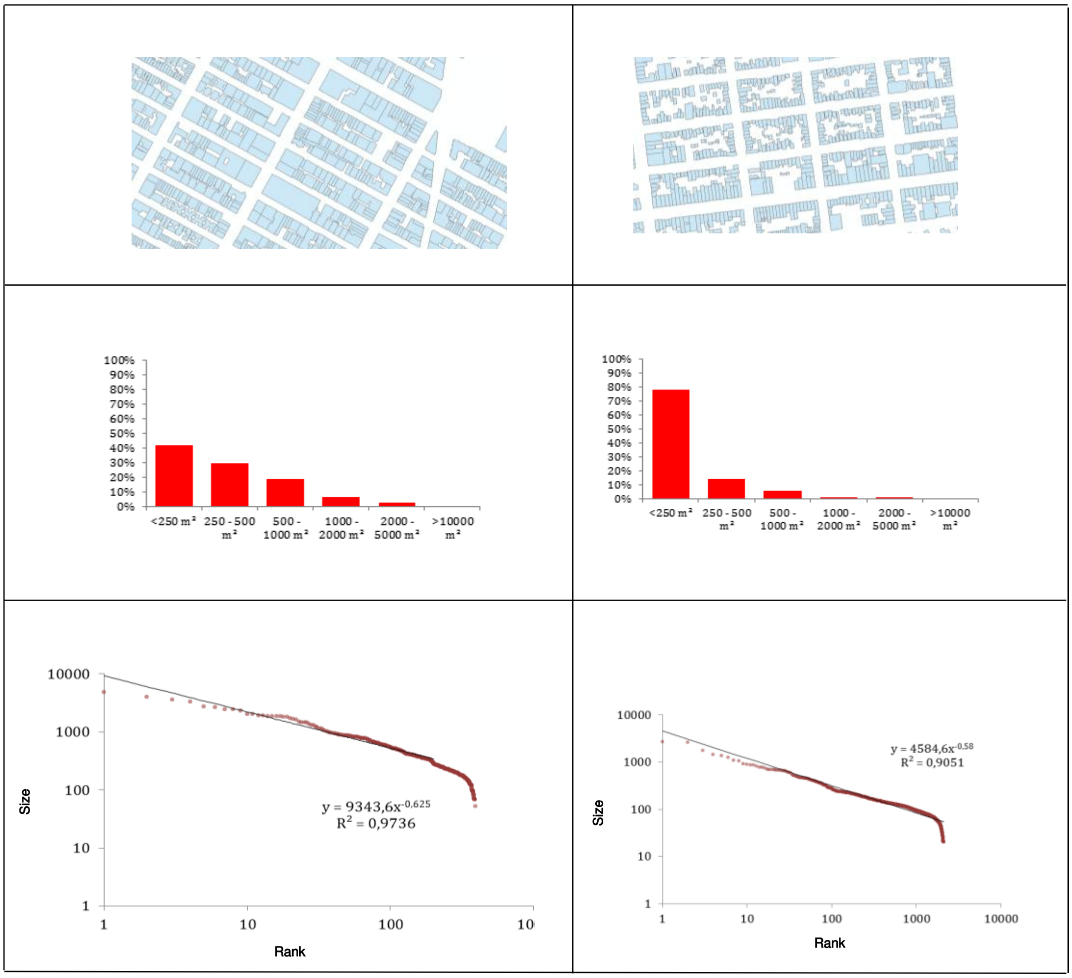


Figure 12 Manhattan land lots around Madison Square (left) and Brooklyn (right). From identical, small-scale modular units, the emergence recombined the land lots of Manhattan to create a hierarchy of scale. An analysis of rank/size (in m^2 and in logarithmic units): left, Madison Square; right, Brooklyn (source: Loeiz Bourdic, Institute of Urban Morphologies and Complex Systems, 2014)

The rank/size analysis of the land lots (Figure 12, shown here on a logarithmic scale in which the slope corresponds to the exponent of the inverse power law) show a fractal universality between a highly developed Madison Square and Brooklyn, in which 80% of the lots still have the same shape and size today as at the beginning of the 19^{th} century. Of course Madison Square has increased its hierarchy but this takes place as if the class of universality of New York were characterised by an exponent of the order of -0.6, with the exception of Lower Manhattan, the oldest and most irregular part, the one developed before the Commissioners' Plan. As we saw in Figure 2, the higher the scaling exponent, the more the gradient between the higher values and the long tail of weaker values slopes: in other terms, the more the distribution is unequal.

Property in Manhattan and in Brooklyn is distributed in a more unequal manner (its structure of scale is more marked) than in Lower Manhattan or in Paris. Is this due to a greater inequality caused by stronger competing market forces in New York than in Paris?

If we look at maps and histograms, we can see that this is not the case. Market forces have transformed Manhattan much more than Brooklyn but they show the same hierarchy of scale. Around Madison Square, 40% of the lots are those of the early 19th century, while Brooklyn is virtually unchanged since the age of Henry James, with 80% of its lots dating back to the 19th century.

The system of lots in Brooklyn and Manhattan on the grid of the Commissioners' Plan, while having undergone different evolutions, adopts a similar geometry that has the same fractal dimension. So how can we explain that Wall Street, the world finance capital, Rue Mouffetard in Paris, whose land lots date back to Philippe II, and Hong Kong with its 19th century street layout, belong to the same class of fractal universality with an exponent of 0.5 difference from that of the Commissioner's Plan grid? Geometry once again. Urban surfaces with an exponent of 0.5 are present in irregular street networks that do not have the geometric superstructure of the blocks made up of regular rectangles like those of Madison Square or Brooklyn. Wall Street dominates world finance from a winding street mapped out by Dutch immigrants in the 17th century.

It therefore appears that the Euclidian chessboard increases the fractal hierarchy, compared to urban surfaces where irregularity (like in Paris or Lower Manhattan) is not enclosed in an orthogonal geometry. On the other hand, where the land lots are not enclosed in a Euclidian grid as in Lower Manhattan (Figure 13) which still shows the virtually unchanged outlines of the streets of New Amsterdam, we find a class of fractal universality with an exponent – ½, which characterises inner city Paris and Hong Kong, in irregular land lots that have evolved organically through multifractal imbrication.

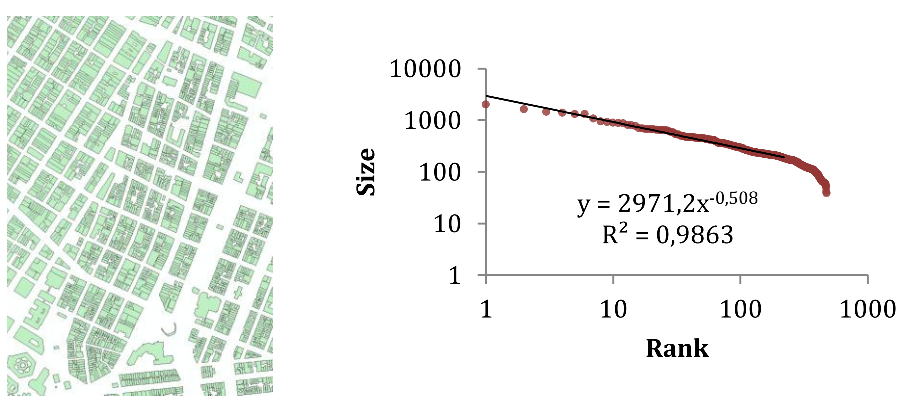


Figure 13 Rank/size analysis (in m^2 and logarithmic units) of the land lots of Lower Manhattan (source: Loeiz Bourdic, Institute of Urban Morphologies and Complex Systems, 2014)

In order to work a city must consume energy. We can construct a concept of energetic density analogous to that of demographic density, that is to say energy consumption divided by the surface on which it is consumed (expressed in Watts per m^2 of urban land, Figure 14). This energy density is a fair approximation of the concentration of activities on urban land. If you look at New York from a distance, for example, two large areas have the greatest concentration of energetic density, which are obviously the two CBDs of Midtown and Lower Manhattan, while Long Island and Brooklyn have a lower energy density.

However, if we enlarge the map and the urban blocks appear, we can see that the urban surface is differentiated with a wide variety of blocks in the middle of dominant area. If we enlarge it again, now the blocks themselves are differentiated. The 'energetic surface' of New York shows a mix of all scales of regularity and irregularity characteristic of multifractals. A rank/size analysis of energy consumption for heating buildings in New York reveals this complexity, like a 'signature'. It leads to the reappearance of an exponent of 0.5 for the Pareto exponent of energy consumption for buildings in New York.

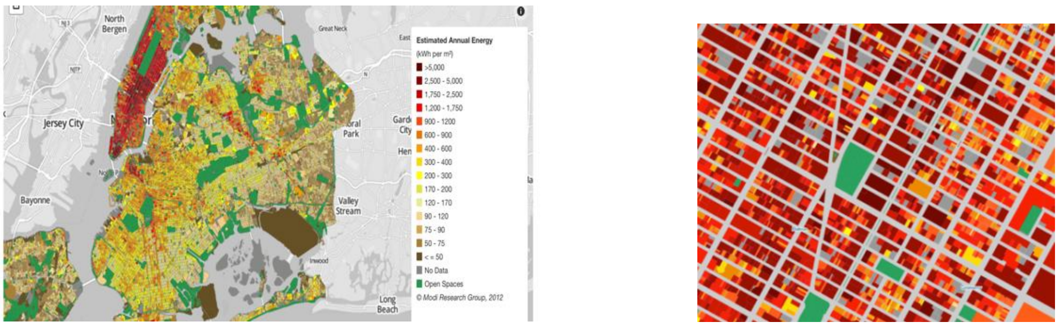


Figure 14 Energetic density (in Watts/m^2) of lots in New York. Left, the whole of New York; right, around Madison Square. Source of maps: Spatial distribution of urban building energy consumption by end use B. Howard, L. Parshall, J. Thompson, S. Hammer, J. Dickinson, V. Modi, 2011

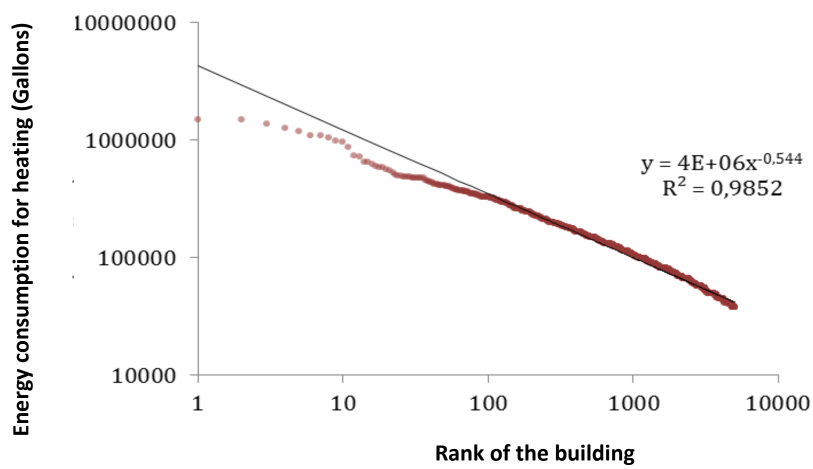


Figure 15 Rank/size analysis of energy consumption for heating in buildings with collective boilers in New York (Source: Loeiz Bourdic, Institute of Urban Morphologies and Complex Systems, 2014)

URBAN SCALING LAWS TO UNDERSTAND THE PAST AND TO BUILD THE FUTURE

An analysis of urban surfaces is only one example of intra-urban complexity ordinated by fractal mathematical regularities. The work of urban geography – in particular that of Denise Pumain – has long shown the presence of rank/size and hierarchy of scale laws in the systems of cities while constructing an evolutionary theory to explain them. Our work allows us to extend these results to all scales of the urban structure itself. It is not only the size of cities that show a hierarchy of scale, but the finer scale of the urban texture, the division into lots, that presents this mathematical regularity. Our earlier work has shown that the size of public parks in Paris, as in Manhattan, is equally hierarchically organised by scale laws, this time to maximum accessibility with a minimum surface area, as in other fractal phenomena studies by physics. The same applies to the street patterns in Paris where the incidence (the cumulative length) of different types of streets (from the Haussmann boulevards to the narrow, winding streets of the Middle Ages) follows a scale law. Other works, in particular those of Sergio Porta, Paolo Crucitti and Vito Latora have shown that if one constructs a 'dual graph' of the streets (*i.e.* if one considers the streets as entities and their intersections as links) and if one applies the techniques of analysis reserved for social networks to these graphs, one finds, especially in complex cities like Ahmedabad or Venice, properties of a hierarchy of scale of the degrees of nodes (*i.e.* the number of intersections per streets), characteristic of the connectivity of complex patterns, sometimes natural like those of the brain or artificial like those of the Internet. The street systems are therefore as scaling in their metric properties (the incidence of different street geometries) as in their topological organisation (the connectivity of streets amongst themselves). This is also true of the number of lines per station and the volumes of passengers who follow this hierarchy of scale in the underground systems of Paris or London, as Loeiz Bourdic demonstrated in his body of work for the Institute of Urban Morphologies and Complex Systems. Finally, it is also true of the population and employment density (as we have shown in numerous European cities), and energy consumption, which obey intra-urban scale laws on extremely fine meshes.

As in natural fractals, this presence of a hierarchy of scale in numerous intra-urban phenomena is a result of the evolutionary selection of the most efficient and most resilient structures. For reasons of space, we refer you to our earlier works.

These results offer a new view of understanding cities on all scales, those of complexity and hierarchy of scale. They also allow us to improve their management. In particular, we have shown that use of transport in cities depends far less on their average density than on the Pareto exponent of this density. In the same way we have improved the models of nodal and local value, initially developed by Luca Bertolini, to explain the development of the underground systems. Our new model combines the hierarchy of scale of the distribution of economic density with the network effects of connectivity. We are already applying it to strategic planning in Shanghai in 2050 with the World Bank and the Shanghai Development and Reform Commission. We are also applying this model in an operational manner to the planning of Chinese cities around the 6000 underground stations that China will have in 2020. The numerous intra-urban scale laws that we have discovered are therefore a means of understanding the past, but also of building the future.

References

Ballon H. (2012), *The Greatest Grid - The Master Plan of Manhatan 1811-2011*, New York, Museum of the City of New York and Columbia University Press.

Noizet H., Bove B. (2013), Costa L., *Paris, de parcelles en pixels: Analyse géomatique de l'espace parisien médiéval et moderne*, Paris, Coédition PU Vincennes

Pumain D., Paulus F., Vacchiani-Marcuzzo C., Lobo C., «An evolutionary theory for interpreting urban scaling laws», *Cybergeo : European Journal of Geography* [En ligne], Systèmes, Modélisation, Géostatistiques, document 343, mis en ligne le 05 juillet 2006, consulté le 12 octobre 2013.

Salat S. (2011), *Les villes et les formes : Sur l'urbanisme durable*, Paris, CSTB et Hermann.